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Book Review: *Introduction to Octionion and Other Non-Associative Algebras in Physics*

Introducion to Octionion and Other Non-Associative Algebras in Physics. Susumu Okubo, Montroll Memorial Lecture Series in Mathematical Physics, 2, Cambridge University Press, 1995.

As stated on its frontispiece, the aim of this book is "to familiarize researchers and graduate students in both physics and mathematics with the application of non-associative algebras in physics." And it does precisely that. This is a clear, brief, and authoritative introduction to the ongoing search for natural applications of octonions to physics, an effort being made by a small number of enthusiastic devotees. Apart from this book, I know of no other introduction to the subject. As such, the book fills a gap in the literature of physics. Professor Okubo is a well-known mathematical physicist, and the book has a clear-headed, right-to-the-point, no-digression style that is reminiscent of Dirac's books, particularly his book on general relativity.

The bulk of the book contains an introduction to, and a listing of, the basic definitions and properties of various algebraic structures. As such, it serves as a convenient reference work. In addition, each chapter makes a connection with a subject in physics in the form of a remark or an introduction to an application.

Chapter 1 sets the scene: starting from the known definitions of complex numbers and their properties, onto those of quaternions, it proceeds from there onto octonions and their non-associative algebras. The non-associativity of octionion algebras, and the possible physical implications of that fact, are central themes of the book.

Chapter 2 is a discussion of the basics of non-associative algebras in general. Chapter 3 is devoted to Hurwitz algebras: composition algebras that include precisely those of the real numbers, the complex numbers, quaternions, and octionions, and as such play a unifying role. Chapter 4 contains a discussion of para-Hurwitz algebras: basically Hurwitz algebras, but without a unit element.

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Chapter 5 is an introduction to real division algebras (real quaternionic and real octonion algebras are examples of real division algebras) and real Clifford algebras (the Majorana–Weyl spinors of superstring theory satisfy a real Clifford algebra). Chapter 6 is devoted to Clebsch–Gordon (angular momentum) algebras as familiar structures that happen to be also nonassociative.

Chapter 7 is devoted to a discussion of the algebra of observables in quantum mechanics. Can one formulate a mathematically consistent and physically viable quantum mechanics on the basis of a nonassociative algebra of observables? Such questions remain open. Chapter 8 is devoted to triple products and ternary systems, as occurs naturally in the Yang–Baxter equations. The idea here is that one can use octionions to obtain new solutions of the Yang–Baxter equations. Some of these solutions already exist in the literature. The physical implications of these solutions are not discussed.

Chapter 9 is devoted to non-associative gauge theory (gauge theories that contain a noncanceling anomaly that leads to a breakdown of the Jacobi identity, and hence of the associativity property). What are the physical properties of these theories? What new physical predictions do they offer? Such questions also remain open. Finally, Chapter 10 is a one-page listing of undiscussed issues.

Octonions are elegant mathematical structures that remain to find a profound application in physics. The author of the book clearly hopes that some day they will. The book is an excellent introduction to the mathematical aspects of the subject; all the definitions are there, gradually introduced, and written in an orderly style. Many examples of possible physical applications are given. However, this is not a book on physics, and a detailed discussion of the physical consequences of a non-associative physics is clearly outside of the scope of a book of this size.

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